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HACking at Non-linearity: Evidence from Stocks and Bonds

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Abstract:

The implicit assumption of linearity is an important element in empirical finance. This study presents a hypothesis testing approach which examines the linear behaviour of the conditional mean between stock and bond returns. Conventional tests detect spurious non-linearity in the conditional mean caused by heteroskedasticity and/or autocorrelation. This study re-states these tests in a heteroskedasticity and autocorrelation consistent (HAC) framework and we find that stock and bond returns are indeed linear-in-the-mean in both univariate and bivariate settings. This study contends that previous research may have detected spurious non-linearity due to size distortions caused by heteroskedasticity and autocorrelation, rather than the presence of genuine non-linearity.

JEL Classifications: G00; G12; G14

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autocorrelation-robust tests

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Introduction

The unstated assumption of linear asset returns is an important condition in empirical finance which receives little research attention. Finance theory provides little or no guidance to the *a priori* expectation of linearity. Whilst the assumption of linearity is not a necessary condition in *finance theory*, it is an important element in *empirical finance*. A linear conditional mean is an important element in empirical finance as the adequacy of portfolio and asset pricing models such as Markowitz (1952), Sharpe (1964), Ross (1976) and Fama and French (1992, 1993) rely on the linearity-in-the-mean assumption. If the conditional mean in asset returns is a non-linear function, then empirical finance models may require re-specification in a more complex non-linear framework. The importance of linearity in empirical finance motivates us to examine the conditional mean in the two most important asset classes, stocks and bonds.[1]

Granger and Teräsvirta (1993) define returns as linear when a model that describes these returns exhibits a linear conditional mean with constant error disturbances over time. Whilst considerable research has examined the time variation of error disturbances (ie. conditional heteroskedasticity effects), the conditional mean in asset returns has received less research attention.[2] The first objective of this study is to examine the conditional mean of stock and bond returns in a univariate setting.[3] Given the importance of stock and bonds to investors, we know surprisingly little about the linear conditional mean behaviour of the monthly returns of these assets in a univariate setting. An important element of this study is to control of the stylised features of heteroskedasticity and autocorrelation in linearity-in-the-mean hypothesis testing. Granger and Teräsvirta (1993) and Lee, White and Granger (1993) caution that error disturbances which are not independent and identically distributed (i.i.d.) may cause erroneous results in tests of linearity-in-the-mean. To more accurately examine the behaviour of the conditional mean, this study proposes a heteroskedasticity and autocorrelation consistent (HAC) approach to control these effects in linearity-in-the-mean tests so that robust statistical inference can be made. Previous studies have not jointly controlled for heteroskedasticity and autocorrelation in linearity-in-the-mean tests which may have contributed to spurious non-linearity detected in the conditional mean. This is the first known empirical study to explicitly control *both* heteroskedasticity and autocorrelation in linearity-in-the-mean tests in monthly stock and bond returns.

Using the HAC approach, we find that stock and bond returns are linear-in-the-mean in a univariate setting, and thus challenges the findings of previous studies from Hsieh (1991), Opong, Mulholland, Fox and Farahmand (1999) and Yadav, Paudyal and Pope (1999). We demonstrate that conventional linearity-in-the-mean tests have a tendency to detect spurious non-linearity caused by the heteroskedasticity and autocorrelation in the error disturbances which contaminates the underlying hypothesis tests. By jointly controlling *both* heteroskedasticity and autocorrelation in the hypothesis testing framework, we demonstrate that stock and bond returns are indeed linear-in-the-mean in a univariate setting.

The second objective of this study is to examine the bivariate relationship between stocks and bond returns. Rational agents making investment decisions in a conventional Markowitz (1952) framework assume that asset returns are unconditionally linearly associated with each other. If asset returns are not linear-in-the-mean in a bivariate setting, then mean-variance investors may require more complex portfolio selection techniques. This study shows that conventional linearity-in-the-mean tests detect erroneous non-linearity between stocks and bonds, however, the HAC tests reveal that stock and bond returns are linear-in-the-mean in a bivariate setting when heteroskedasticity and autocorrelation are controlled. The bivariate results from this study challenge the findings of Boudoukh, Richardson and Whitelaw (1997) and Desai and Bharati (1998). We contend that the difference between our findings and those of previous studies is due to the lack of control of the *joint* effects of heteroskedasticity and autocorrelation in previous research. Even when previous studies such as Hsieh (1991), Opong *et. al.*, (1999), Poshakwale (2002) and Yadav *et. al.*, (1999) partially control for heteroskedasticity, they do not control for autocorrelation in the test residuals. Hence, previous studies have failed to control the joint effects of *both* heteroskedasticity and autocorrelation which has led to the spurious conclusion that non-linearity in the conditional mean is present in the bivariate relationship between stock and bond returns.

A number of issues arise from this study. First, we show that the univariate and bivariate behavior of stock and bond returns is linear-in-the-mean, while the non-linearity detected in previous studies is due to the size distortions in the error disturbances. The consequence of this finding suggests that researchers should direct their research attention towards the refinement of linear-based models that accommodate the dynamic behaviour of error disturbances rather than the development of specific non-linear models.[4] Second, this study highlights the pronounced effects of heteroskedasticity and autocorrelation on tests of linearity-in-the-mean. By highlighting these effects in this study, researchers can more readily understand the role that these empirical features play in the future development of portfolio selection and asset pricing frameworks. Finally, the empirical findings from this study support the concern in Granger and Teräsvirta (1993) and Lee *et. al.,* (1993) regarding the impact of error disturbances on the efficacy of linearity-in-the-mean tests.

The rest of the study is organized as follows. In Section 2 we provide a review of the related literature. Section 3 documents the methods employed to examine the assumption of linearity-in-the-mean, with Section 4 describing the data employed in this study. Section 5 examines the results while Section 6 offers concluding remarks.

Related Literature

Finance theory does not explicitly impose the assumption of linearity-in-the-mean on asset returns, however a number of theoretical rationales have been proposed in the microstructure literature to explain the presence of non-linear behaviour. The first theoretical rationale to justify non-linear returns comes from studies that examine market equilibrium in the presence of transaction costs and market frictions. Dumas (1992), He and Modest (1995) and Sercu, Uppal and Van Hulle (1995) suggest that transaction costs and market frictions give rise to small deviations in asset prices which result in partial mispricings from market equilibrium. They argue that these misalignments persist until the size and deviation of the mispricing is large enough for arbitrageurs to enter the market and cause a non-linear adjustment of prices back to equilibrium. Although this theoretical construct is valid in a microstructure setting, it is less supportive in a portfolio selection framework whereby lower frequency samples such as monthly returns are examined.

The second theoretical rationale to explain non-linearity in asset returns exists in the absence of transaction costs. Black and McMillan (2004) and McMillan (2005) argue that the behavioural finance concept of cognitive biases in investment behaviour may not be consistent with expected utility maximization and thus non-linear asset price deviations may result.

The third rationale which may explain non-linear returns comes from Shleifer and Vishny (1997) who argue that the limits of arbitrage may be ineffective in extreme circumstances. During extreme market conditions, capital constraints by arbitrageurs may result in non-linear deviations of asset prices from their true value. The reversal of these market inefficiencies occur when arbitrageurs believe that price misalignments are at levels where mean reversion strategies can be rewarded.

Overall, the theoretical finance literature provides a number of possible explanations to justify non-linear asset return behaviour in the microstructure setting, however, little theoretical guidance exists for long-term investors examining the linearity-in-the-mean of monthly asset returns in a portfolio selection framework. The focus of this study is in this low-frequency setting of monthly returns.

Whilst the literature provides very little theoretical guidance for the presence of non-linear returns for long-term investors, the linear behaviour of asset returns can be examined in an empirical setting. The importance of examining the linearity of asset returns cannot be overstated as this implicit assumption is paramount in empirical portfolio and asset pricing frameworks. A researcher's proposal to consider a non-linear empirical model must be reminded of Granger and Teräsvirta (1993) who caution the use of non-linear models without first testing for non-linearity. Thus, the decision to choose between a linear or non-linear model to explain asset returns is of primary importance. It therefore seems logical that the linear dependence of asset returns be empirically tested in order to avoid model mis-specification in portfolio selection and asset pricing.

In the econometrics and statistics literature, many tests have been developed to examine the linear behaviour of variables. The Ramsey (1969) Regression Specification Reset Test (RESET) was one of earliest tests which detects non-linearity in the functional form of a linear model. The Ramsey (1969) framework was then re-specified in Keenan (1985) in a more simplified framework to avoid multicollinearity. The Keenan (1985) test was extended by Tsay (1986) and Teräsvirta, Lin and Granger (1993) to examine multiplicative forms of linearity-in-the-mean by employing quadratic, cubic and cross-product terms. To examine the linear behaviour of the error disturbances of linear models, the seminal works of Engle (1982) and Bollerslev (1986) developed conditional heteroskedasticity based frameworks. As a more general test of linearity, Brock, Dechert and Scheinkman (1987) developed the BDS test to examine the i.i.d. assumption in a time series. As a specific form of non-linear testing, Lo (2001) segregates asset returns into up and down regressors which examine if the beta coefficients are statistically significant. Other studies including Boudoukh *et. al.,* (1997) and Mitchell and Pulvino (2001) employ piecewise regression frameworks to evaluate non-linearity. These research contributions represent some of the many linearity tests developed in the literature, however, many more exist which are outside the scope of this study. The most striking feature in the linearity literature is the loosely defined term of non-linearity and the multiple tests that have been developed to identify and detect various forms of it.

Some of these linearity tests have been applied in an empirical setting to evaluate the linear behaviour of stock and bond returns and they can be divided into two strands of literature, namely univariate and bivariate tests. In the univariate setting, researchers have examined the autoregressive (AR) process of asset returns to consider whether current returns can be explained by the non-linear behaviour of past returns. The key stockmarket literature by Scheinkman and LeBaron (1989), Hsieh (1991), Opong *et. al.,* (1999) and Poshakwale (2002) finds that stock returns in developed and emerging markets are non-linear in the univariate setting. In the interest rate literature, the evidence of univariate linearity is mixed and inconsistent. Ait-Sahalia (1996) and Stanton (1997) find non-linearity in short-rates while Chapman and Pearson (2000) and Jones (2003) challenge these findings.

Despite the scholarly contributions in the univariate framework, little research attention has considered linearity in the bivariate setting. Studies which examine the bivariate setting are important because they consider the linearity assumption between two exogenous variables. These bivariate linearity studies are of particular interest to mean-variance investors as they assume linearity when combining two or more assets in a portfolio selection setting. Boudoukh *et. al.,* (1997) detect non-linearity between the equity risk premium and the term structure by employing a piecewise linear regression model. In another study, Desai and Bharati (1998) detect non-linearity between stock and bond returns by employing a variety of linearity tests.[5] These research contributions motivate this study to examine the bivariate linear behaviour between stocks and bonds as it relates to a mean-variance investor.

The current state of the literature reveals the following issues that need to be addressed. First, various studies have considered linearity in a univariate setting, however, little research (other than Boudoukh *et. al.*, (1997) and Desai and Bharati (1998)) consider the bivariate linear behaviour between stocks and bonds. The linear behaviour between stocks and bonds is a necessary condition for mean-variance investors, therefore, it is surprising that few studies have investigated this important research question. The paucity of research which considers linearity between stocks and bonds provides the motivation to better understand linearity-in-the-mean from the perspective of a mean-variance investor. Therefore, a large part of this study considers linearity-in-the-mean in a bivariate setting.

Second, the general approach in Granger and Teräsvirta (1993), Campbell, Lo and MacKinlay (1997) and Tsay (2002) shows that any relationship with a non-constant variance (ie. ARCH effects) can be technically defined as non-linear. As heteroskedasticity is a stylised feature of financial market returns, it is our conjecture that research should be concentrated towards the second form of linearity, namely, the linearity of the conditional mean.

Third, whilst specific criticism can be directed toward the various linearity tests in the literature, one specific critique is the loose treatment of heteroskedasticity and autocorrelation effects in the hypothesis tests of past studies. Granger (1993), Granger and Teräsvirta (1993) and Lee *et. al.*,(1993) caution the use of linearity-in-the-mean tests in the presence of heteroskedasticity and autocorrelation as they found that these effects distort the power and robustness of these hypothesis tests. To ensure precise statistical inference, tests for linearity-in-the-mean must be formulated to control for these effects. This study proposes tests of linearity-in-the-mean that can be augmented to explicitly control *both* heteroskedasticity and autocorrelation. To accommodate these research questions, this study comprehensively examines the linearity-in-the-mean in stock and bond returns in both univariate and bivariate settings.

Methodology

This study examines the linearity of the conditional mean by employing the general methodological apparatus from Granger and Teräsvirta (1993), Campbell *et. al.*, (1997) and Tsay (2002). To test for linearity-in-the-mean in a univariate and bivariate setting, a linear relationship is expressed in the following generalized forms:

$$y_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t \tag{1}$$

$$y_t = \varphi_0 + \varphi_1 x_{1t} + \varphi_2 x_{2t} + \ldots + \varphi_i x_{it} + \varepsilon_t \tag{2}$$

where y_t is the return of the dependent variable, x_t is the return of the independent variables, φ_0 is the regression intercept, φ_i are the regression slope coefficients, p is the lag order, ε_t is the random error disturbances and T is the sample size. The univariate model in (1) and the bivariate framework in (2) are considered 'linear-in-mean' when the inclusion of a non-linear parameter in $\varphi_i y_{t-i}$ or $\varphi_i x_{it}$, respectively, results in no statistical improvement in model inference.

As with all estimations in the ordinary least squares (OLS) framework, the statistical inference of the overall model may be susceptible to mis-specification or bias given the behaviour of the error disturbances ε_t. It is possible that non-constant variance effects or serial correlation in ε_t may result in error disturbances which are not i.i.d. Teräsvirta and Granger (1993) and Lee *et. al.*, (1993) remind us that error disturbances which are not i.i.d. may result in the over-rejection of the null hypothesis (ie. Type I error) in linearity-in-the-mean tests. It is therefore imperative that these effects in ε_t be explicitly isolated and controlled from $\varphi_i(\cdot)$ and the underlying linearity-in-the-mean tests.

The general hypothesis test considered in this study can therefore be stated as:

H_0: We cannot reject the null hypothesis of linearity-in-the-mean in $\varphi_i(\cdot)$ after the adjustment for heteroskedasticity and autocorrelation in ε_t.

H_1: At least one non-linear parameter in $\varphi_i(\cdot)$ increases the overall statistical significance of a model after the adjustment for heteroskedasticity and autocorrelation in ε_t.

To examine linearity-in-the-mean whilst controlling heteroskedasticity and autocorrelation in ε_t, this study proposes to employ the following linearity-in-the-mean hypothesis tests: (i) the Keenen (1985) test; (ii) the Tsay (1986) test; and, (iii) the Teräsvirta, Lin and Granger (1993) V23 test.

The Keenan (1985), Tsay (1986) and Teräsvirta *et. al.*, (1993) V23 tests belong to a family of hypothesis tests that approximate general non-linear functions with higher-order combinations of the independent variables. These hypothesis tests employ a restricted least squares approach via an F-test to compare the sum of squared residuals

(SSR) from an original unrestricted model (where $f(x)$ is a quadratic and/or cubic function of x) versus the sum of squared residuals from a simpler model such as (1) or (2). The F-test determines if the model with non-linear functional form has more power than the restricted linear model. A common feature of all of these tests is that they have some power against general non-linear alternatives. The selection of these linearity-in-the-mean tests in this study is motivated by their ability to detect non-linearity and their framework which allows heteroskedasticity and autocorrelation effects in the error terms to be controlled.

Univariate and bivariate test specification

The univariate and bivariate tests of linearity-in-the-mean in this study are specified from an investment based framework. Modern portfolio theory (MPT) serves as the motivation and framework to consider the linearity-in-the-mean in traditional asset classes. The objective of these hypothesis tests is to examine whether stock and bond returns are linear-in-the-mean.

First, we examine the linearity-in-the-mean of asset returns in a univariate framework as asset returns are assumed to satisfy the linearity assumption when they are combined in a portfolio selection framework. Assets returns which reject the null hypothesis of linearity-in-the-mean in a univariate setting may cause spurious results in portfolio selection optimization. Asset returns which reject the null hypothesis of linearity-in-the-mean in a univariate setting may have spillover effects in subsequent bivariate tests. The univariate tests of linearity-in-the-mean serve as a reference prior to the introduction of an exogenous variable in a bivariate linearity-in-the-mean framework.

Second, the bivariate tests of linearity-in-the-mean in this study are specified in a way which reflects portfolio investment behaviour. In a portfolio selection setting, an investor has to consider the investment opportunity set available to determine optimal portfolio choices at time t only. That is, investors in a portfolio selection framework do not have access to asset returns of lagged variables. However, Granger and Teräsvirta (1993) notes that an efficient test for linearity in the econometrics literature includes all possible lagged endogenous and exogenous variables in the linear functional form. The divergent assumptions between MPT and the econometrics literature provide a conundrum. Therefore, to specify these bivariate linearity-in-the-mean tests as it relates in a mean-variance investor, this study specifies these tests to asset returns available at time t only, and is therefore restricted from considering lagged endogenous and exogenous variables from $t-1$, $t-2$, ..., $t-n$ and so forth.[6] We proceed to detail the mathematical specifications of each test.

Keenan (1985) test

The Keenan (1985) test detects model mis-specification of quadratic functional form. In terms of this study, the Keenan (1985) test examines whether a quadratic fitted regression estimate from the original regression improves the statistical significance of the underlying model.[7]

The Keenan (1985) univariate test employs the estimate $\hat{y}_t$ from (1) in the following regression:

$$\hat{y}_t^2 = \varphi_0 + \sum_{i=1}^{p} \varphi_i y_{t-i} + u_t \tag{3}$$

and

$$\hat{\varepsilon}_t = \alpha \hat{u}_t + v_t \tag{4}$$

where $\hat{y}_t^2$ is the fitted squared value of y_t from (1), φ_0 is the regression intercept, φ_i represents the regression slope coefficients, p is the lag order, $\hat{u}_t$ is the random error term estimated in (3) and $\hat{\varepsilon}_t$ is the random error term estimated in (1). The regression in (3) is estimated to remove the linear dependence of $\hat{y}_t^2$ on the regressors in (1). The regression in (4) then employs the estimated residuals from (1) and (3) to form the unrestricted sum of squared errors $SSR_1 = \sum_{t=p+1}^{T} \hat{v}_t^2$. The Keenan (1985) null hypothesis is $H_0: \alpha = 0$ which is given by:

$$F = \frac{(SSR_0 - SSR_1)/g}{SSR_1/(T-p-g)} \quad \text{with } g = s + p + 1 \tag{5}$$

where SSR_0 is the restricted sum of squared errors from (1), s equals the number of powers required greater than one (ie. in this restricted quadratic Keenan test, s equals 2) and F is the F-statistic which is approximately $F(g, T-p-g)$ distributed under the null hypothesis.[8]

In the bivariate setting, the Keenan (1985) test compares the restricted regression in (2) with the following unrestricted regression:

$$\hat{y}_t^2 = \varphi_0 + \varphi_1 x_t + \ u_t \tag{6}$$

and

$$\hat{\varepsilon}_t = \alpha\ \hat{u}_t + v_t \tag{7}$$

Again, $\hat{y}_t^2$ is the fitted squared value of y_t from (2), φ_0 is the regression intercept, φ_1 is the regression coefficient of the single independent variable and $\hat{u}_t$ is the random error term estimated in (6), $\hat{\varepsilon}_t$ is the random error term estimated in (2) and u_t is the random error term. Equation (6) is estimated to remove the linear dependence of $\hat{y}_t^2$ on the single regressor in (6). The sum of squared errors in (7) and (2) are examined under the null hypothesis of $H_0 : \alpha = 0$ which is estimated via an F-statistic which is approximately $F(g, T - p - g)$ distributed under the null hypothesis.

Tsay (1986) test

The second linearity-in-the-mean test employed in this study is the Tsay (1986) framework which extends Keenan (1985) by testing auxiliary regressors which include quadratic and cross-product terms. Not only does the Tsay (1986) test examine non-linearity of quadratic terms, it also evaluates multiplicative terms also which makes it a different and yet more powerful test than Keenan (1985). Lee *et. al.,* (1993) finds reasonable power and robustness from the Tsay (1986) test and it is also regarded as a benchmark test in the linearity literature.

In the univariate setting, the Tsay (1986) test examines if quadratic and multiplicative auxiliary regressors of $\varphi_i(\cdot)$ in (1) are statistically significant. The Tsay (1986) test is given by:

$$y_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{i=1}^{p} \sum_{j=i}^{p} \delta_{ij} y_{t-i} y_{t-j} + v_t \tag{8}$$

where y_t is the return of the dependent variable, φ_0 is the regression intercept, φ_i is the regression coefficients, δ_{ij} is the regression parameter for each auxiliary regressor possessing quadratic and cross-product terms, p is the lag order, v_t is the random error term and T is the sample size. The Tsay (1986) test examines the null hypothesis $H_0 : \delta_{ij} = 0$ against $H_1 : \delta_{ij} \neq 0$ by calculating an F-statistic which is approximately $F(m, T - p - m - 1)$ distributed under the null hypothesis with m auxiliary regressors and $T - p - m - 1$ degrees of freedom.

In the bivariate setting, the Tsay (1986) test is augmented with the restriction to exogenous variables at time *t* only in the following expression:

$$y_t = \varphi_0 + \varphi_1 x_t + \sum_{i=0}^{p} \sum_{j=i}^{p} \delta_{ij} x_{t-i} x_{t-j} + v_t \tag{9}$$

where y_t is the return of the dependent variable, x_t is the return of the independent variable, φ_0 is the regression intercept, φ_1 is the regression coefficient of x_t, δ_{ij} is the regression coefficient of the auxiliary regressor, p is zero, v_t is the random error term and T is the sample size. The Tsay (1986) test examines the null hypothesis $H_0 : \delta_{ij} = 0, \ \forall \ i, j$ against $H_1 : \delta_{ij} \neq 0$ by estimating an F-statistic which is approximately $F(m, T - p - m - 1)$ distributed under the null hypothesis with m auxiliary regressors and $T - p - m - 1$ degrees of freedom.

Teräsvirta, Lin and Granger (1993) V23 test

Although the Tsay (1986) test is highly regarded in the literature, a more powerful test known as the V23 test developed by Teräsvirta *et. al.*, (1993) examines quadratic, cubic and relevant cross-product terms of the independent variables. The V23 test is uniquely different to Keenan (1985) and Tsay (1986) because it considers non-linearity in the form of cubic terms in addition to the quadratic and cross-product auxiliary regressors proposed by Tsay (1986). Simulation studies by Teräsvirta *et. al.*, (1993) demonstrate that the V23 test is a more powerful test in comparison to others when the type of non-linearity is unspecified. This feature of the V23 test makes it the method of choice for testing linearity-in-the-mean despite the little research attention that it has received in the literature.

In a univariate setting, the Teräsvirta *et. al.*, (1993) V23 test examines if the quadratic and cubic terms and multiplicative auxiliary regressors in $\varphi_i(\cdot)$ in (1) are statistically significant. The Teräsvirta *et. al.*, (1993) V23 test is given by:

$$y_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i y_{t-1} + \sum_{i=1}^{p} \sum_{j=i}^{p} \delta_{ij} y_{t-i} y_{t-j} + \sum_{i=1}^{p} \sum_{j=i}^{p} \sum_{k=j}^{p} \delta_{ijk} y_{t-i} y_{t-j} y_{t-k} + v_t \qquad (10)$$

where y_t is the return of the dependent variable, φ_0 is the regression intercept, φ_i represents the regression coefficients, δ_{ij} is the regression parameter for each auxiliary regressor possessing quadratic and cross-product terms, δ_{ijk} is the regression parameter for each auxiliary regressor possessing cubic and cubic-based cross-product terms, p is the lag order, v_t is the random error term and T is the sample size. The Teräsvirta *et. al.*, (1993) V23 test examines the null hypothesis $H_0: \delta_{ij} = \delta_{ijk} = 0, \ \forall \ i, j, k$ against $H_1: \delta_{ij} \neq 0$ or $\delta_{ijk} \neq 0$ as an F-statistic which is approximately $F(m, T-p-m-1)$ distributed under the null hypothesis with m auxiliary regressors and $T-p-m-1$ degrees of freedom.

In the bivariate setting, the Teräsvirta *et. al.*, (1993) V23 test is re-specified to examine the linearity-in-the-mean with a single exogenous variable. The bivariate Teräsvirta *et. al.*, (1993) V23 test can be expressed as:

$$y_t = \varphi_0 + \varphi_1 x_t + \sum_{i=0}^{p}\sum_{j=i}^{p}\delta_{ij}x_{t-i}x_{t-j} + \sum_{i=0}^{p}\sum_{j=i}^{p}\sum_{k=j}^{p}\delta_{ijk}x_{t-i}x_{t-j}x_{t-k} + v_t \tag{11}$$

where y_t is the return of the dependent variable, x_t is the return of the independent variable, φ_0 is the regression intercept, φ_1 is the regression coefficient, p is zero, δ_{ij} is the regression parameter for each auxiliary regressor possessing quadratic and cross-product terms, δ_{ijk} is the regression parameter for each auxiliary regressor possessing cubic and cubic-based cross-product terms, v_t is the random error term and T is the sample size. The Teräsvirta *et. al.*, (1993) V23 test examines the null hypothesis $H_0: \delta_{ij} = \delta_{ijk} = 0, \ \forall \ i, j, k$ against $H_1: \delta_{ij} \neq 0$ or $\delta_{ijk} \neq 0$ by examining an F-statistic which is approximately $F(m, T-p-m-1)$ distributed under the null hypothesis with m auxiliary regressors and $T-p-m-1$ degrees of freedom.

Controls for heteroskedasticity and autocorrelation

To control for heteroskedasticity and autocorrelation in the Keenan (1985), Tsay (1986) and Teräsvirta *et. al.*, (1993) V23 tests, this study proposes that the F-statistic be re-specified as a set of augmented Wald tests which are heteroskedasticity and autocorrelation consistent (HAC). The Wald statistics for these hypothesis tests can be mathematically expressed as:

$$W_W = T(\hat{\theta}^R - 0)'\hat{\Omega}_W^{-1}(\hat{\theta}^R - 0) \tag{12}$$

$$W_{NW} = T(\hat{\theta}^R - 0)'\hat{\Omega}_{NW}^{-1}(\hat{\theta}^R - 0) \tag{13}$$

where W_W is the White (1980) heteroskedasticity-consistent (HC) Wald Statistic of the respective test, $\hat{\Omega}_W^{-1}$ is the White (1980) heteroskedasticity-consistent sample covariance matrix from the residuals derived from the respective test, $\hat{\theta}^R$ is the vector of regression estimators from the respective test, W_{NW} is the Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) Wald statistic of each test, and $\hat{\Omega}_{NW}^{-1}$ is the Newey and West (1987) HAC sample covariance matrix from the residuals derived from each test. This study will emphasize the linearity-in-the-mean hypothesis test estimates derived from the Newey and West (1987) HAC Wald statistic as these results examine linearity-in-the-mean whilst controlling for the joint effects of heteroskedasticity and autocorrelation.

Data

The data employed in this study consists of global and U.S. index returns. We employ continuous compounded excess returns of various stock and bond indexes consisting of 144 monthly observations for the twelve year period from January 1994 to December 2005. Monthly excess returns are employed in this study as we are motivated to examine the linear behaviour between stock and bond returns in a finance framework as it relates to a mean-variance investor.

The Morgan Stanley Commodity Index (MSCI) All Country World Equity Index is employed as a proxy for world stock returns. To replicate U.S. stock returns, we utilize the Standard and Poors (S&P) 500 All Return Index and the MSCI U.S. Equity Index. To better understand the linearity-in-the-mean and the variation of stock returns, we also employ the Fama and French (1992, 1993) SMB and HML factors and the Carhart (1997) UMD risk factor for comparative purposes.[9]

To proxy global bond returns, we employ the Morgan Stanley (MS) World plus Emerging Sovereign Bond Index, the J.P. Morgan Global Government Bond Index and the Lehman Global Aggregate Index. For US bond returns, we utilize the Morgan Stanley US Government Bond Index and the Lehman US Aggregate Index. The risk-free rate employed in this study is the Ibbotson and Associates U.S. 1 month Treasury Bill rate.

The summary statistics in Table 1 clearly reflect the salient features of stock and bond returns. The empirical characteristics of negative skewness, excess kurtosis and non-normality in most stock and bond index returns are the dominant features in the data. Another striking feature is the statistically significant serial correlation in the second moment (ie. non-constant variance) in stock returns.[10] In contrast, world bond returns exhibit statistically significant serial correlation while U.S. bonds report significant second order negative correlation.

Overall, Table 1 highlights the serial correlation in the first and second moments in returns which may affect the inference of the linearity-in-the-mean tests employed in this study. It is clear that the linearity-in-the-mean hypothesis tests will be estimated in the presence of heteroskedasticity and serial correlation in the data. We proceed to detail the results of the various linearity-in-the-mean hypothesis tests.

Table 1: Summary Statistics

This table presents the summary statistics of the monthly excess returns of the stocks and bond indexes employed in this study. We also include the Fama-French (1992,1993) and Carhart (1997) risk factors for comparative purposes also. Panel A shows the descriptive statistics of the monthly excess returns of the respective indices. Panel B reports the autocorrelation of returns. Panel C shows the autocorrelation of squared returns. Panel D reports the normalized z-scores of the 1st, 2.5th, 5th, 95th, 97.5th and 99th percentiles. The 1%, 2.5%, 5%, 95%, 97.5% and 99% percentiles for a normal distribution are -2.3263, -1.9600, -1.6449, 1.6449, 1.9600 and 2.3263, respectively. The data is sampled monthly from January 1994 to December 2005 consisting of 144 observations. * and ** denote statistical significance at the 5% and 1% levels, respectively.

	Stocks						Bonds				
Sector	World	USA	USA	USA	USA	USA	World	World	World	USA	USA
Variable	MSCI World Index	S&P500 Index	MSCI USA Equity Index	HML	SMB	UMD	MSCI World plus Em. Sov Index	JP Morgan Global Govt Bond Index	Lehman Global Aggregate Index	MSCI US Govt. Bond Index	Lehman USA Aggregate Index
Panel A: Descriptive Statistics											
Mean	0.360	0.521	0.531	0.493	-0.199	0.698	0.164	0.235	0.219	0.183	0.317
Standard Deviation	4.028	4.280	4.318	3.615	4.182	5.393	1.897	0.900	0.884	1.341	2.524
Skewness	-0.754	-0.746	-0.690	0.291	-1.655	-1.216	0.353	-0.275	-0.348	-0.423	-0.556
Kurtosis	4.002	3.967	3.732	4.890	12.140	9.713	3.328	3.335	3.412	3.718	4.139
Median	0.796	1.085	1.100	0.444	-0.200	0.871	0.350	0.310	0.317	0.245	0.584
Maximum	8.455	8.900	9.100	12.848	12.628	16.890	5.490	2.970	2.986	3.520	6.991
Minimum	-14.696	-16.020	-15.370	-10.336	-24.680	-28.835	-4.480	-2.410	-2.184	-4.660	-9.714
Jarque-Bera Statistic	19.680	18.967	14.622	21.911	547.384	294.454	3.635	2.483	3.926	7.377	15.194
Jarque-Bera p-value	0.000**	0.000**	0.000**	0.000**	0.000**	0.000**	0.162	0.289	0.140	0.025*	0.001**
Panel B: Autocorrelation (First Moment)											
AC1	0.016	-0.016	-0.014	0.134	0.185*	-0.072	0.203*	0.190*	0.184*	0.087	0.072
AC2	-0.035	-0.027	-0.019	0.019	0.017	-0.099	0.018	0.039	0.002	-0.156*	-0.193**
AC3	0.055	0.066	0.087	0.039	-0.203*	0.028	0.064	0.129	0.126	0.079	0.073
AC6	0.111	0.086	0.090	0.019	0.077	0.184*	-0.031	-0.035	0.006	-0.047	-0.028
AC12	0.086	0.081	0.091	0.109	0.109	0.195*	-0.041	-0.170	-0.174*	-0.093	-0.106
Panel C: Autocorrelation (Second Moment)											
AC1	0.035	0.091	0.107	0.314**	0.427**	0.185*	-0.037	0.036	0.035	-0.036	-0.041
AC2	0.201*	0.181**	0.181*	0.403**	0.118	0.114	-0.017	-0.021	-0.038	0.134	0.155
AC3	0.059	0.123	0.160	0.465**	0.174*	0.053	-0.050	-0.034	-0.002	-0.017	-0.050
AC6	0.089	0.115	0.109	0.132	-0.025	0.068	-0.002	-0.041	-0.064	-0.061	-0.083
AC12	0.131	0.088	0.095	0.334**	0.006	0.038	-0.056	-0.043	-0.049	0.006	-0.055

Results

The results of this study are presented in two parts, namely, univariate and bivariate tests. To demonstrate the effects of heteroskedasticity and autocorrelation, this study presents three p-values for each test. The first p-value is calculated from the conventional linearity-in-the-mean test. The second p-value is estimated from the White (1980) heteroskedasticity consistent (HC) Wald test while the third and final p-value is calculated from the Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) Wald test.

Two key findings can be drawn from these results. First, the conventional linearity-in-the-mean tests detect spurious non-linearity when examining various monthly stock and bond returns. Second, when the joint effects of heteroskedasticity and autocorrelation in the error disturbances are controlled, we find that stock and bond returns are actually linear-in-the-mean in both univariate and bivariate settings.

Univariate results

Table 2 presents the p-values from the univariate Keenan (1985), Tsay (1986) and Teräsvirta e*t. al.*, (1993) tests for autoregressive models of first, second and third order. The Keenan (1985) tests in Table 2 report infrequent statistically significant p-values. In contrast, the more powerful Tsay (1986) and Teräsvirta *et. al.*, (1993) V23 tests in Table 2 report statistically significant p-values. The second p-value presented for each test is the HC Wald test and it generally reports statistically significant p-values. The third p-value of each test is the HAC result which shows that for all tests, we cannot reject the null hypothesis of linearity-in-the-mean.

The results in Table 2 provide overwhelming empirical evidence to suggest that non-linearity detected in conventional univariate tests is due to the effects of both heteroskedasticity and autocorrelation in the error disturbances rather than genuine non-linearity in the conditional mean.[11] The results also suggest that, at times, the HC Wald tests also provide distorted test statistics due to autocorrelation in the error disturbances. This effect can be readily seen in world bond index returns whereby the HC p-values report statistically significant p-values caused by the serial correlation in the error disturbances. When the HAC hypothesis tests are estimated, we discover that all p-values are statistically insignificant. These results support the findings of Granger and Teräsvirta (1993) and Lee *et. al.*, (1993) who caution that heteroskedasticity and autocorrelation effects can distort linearity-in-the-mean hypothesis tests. The conclusions to be drawn from Table 2 clearly demonstrate that the asset returns employed in this study are indeed linear-in-the-mean in a univariate setting.

Table 2: Univariate Linearity-in-the-Mean Tests

This table reports the p-values of the Keenan (1985), Tsay (1986) and the Teräsvirta *et. al.,* (1993) V23 tests in a univariate setting with lag orders of one, two and three, respectively. Three p-values are estimated for each test. The first p-value is estimated from the conventional test. The second p-value is from the test adjusted as a Wald test employing a White (1980) heteroskedasticity-consistent (HC) covariance matrix. The third p-value is from the test adjusted as a Wald test employing a Newey-West (1987) heteroskedasticity and autocorrelation consistent (HAC) covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

	Test Types								
Variable	Keenan AR(1)	Keenan AR(2)	Keenan AR(3)	Tsay AR(1)	Tsay AR(2)	Tsay AR(3)	V23 AR(1)	V23 AR(2)	V23 AR(3)
Panel A: Stocks									
MSCI World Equity Index	0.959	0.977	1.000	0.529	0.410	0.478	0.721	0.361	0.271
	0.705	0.840	0.999	0.514	0.684	0.491	0.785	0.334	0.000**
	0.793	0.893	0.999	0.508	0.747	0.549	0.753	0.931	0.815
S&P 500 All Return Index	0.925	0.715	1.000	0.406	0.304	0.350	0.749	0.281	0.269
	0.630	0.378	1.000	0.468	0.587	0.135	0.581	0.004**	0.000**
	0.816	0.726	1.000	0.552	0.787	0.558	0.721	0.874	0.065
MSCI USA Equity Index	0.930	0.798	1.000	0.422	0.376	0.365	0.477	0.236	0.323
	0.678	0.498	1.000	0.503	0.695	0.108	0.650	0.002**	0.997
	0.833	0.766	1.000	0.579	0.822	0.532	0.742	0.863	0.998
HML	0.153	0.263	0.869	0.001**	0.034*	0.025*	0.026*	0.006**	0.022*
	0.070	0.164	0.734	0.036*	0.097	0.007**	0.004**	0.001**	0.003**
	0.333	0.540	0.833	0.192	0.514	0.824	0.379	0.794	0.888
SMB	0.004**	0.012*	0.064	0.000**	0.003**	0.015*	0.000**	0.043*	0.033*
	0.000**	0.000**	0.004**	0.000**	0.000**	0.000**	0.000**	0.000**	0.998
	0.409	0.645	0.748	0.323	0.601	0.846	0.606	0.745	0.999
UMD	0.941	0.807	0.981	0.457	0.465	0.164	0.015*	0.043*	0.082
	0.725	0.714	0.956	0.491	0.792	0.088	0.000**	0.000**	0.000**
	0.782	0.582	0.898	0.581	0.691	0.653	0.330	0.738	0.999
Panel B: Bonds									
MSCI World plus Em.Sov. Index	1.000	1.000	1.000	0.956	0.485	0.528	0.414	0.457	0.363
	0.962	1.000	0.971	0.833	0.684	0.628	0.343	0.664	0.786
	0.974	1.000	0.982	0.840	0.691	0.664	0.513	0.898	1.000
JP Morgan Global Govt Bond Index	0.237	0.237	0.561	0.004**	0.081	0.210	0.048*	0.327	0.424
	0.012*	0.012*	0.064	0.038*	0.147	0.390	0.110	0.379	0.124
	0.139	0.263	0.337	0.091	0.361	0.692	0.223	0.851	0.971
Lehman Global Aggregate Index	0.211	0.254	0.749	0.003**	0.089	0.284	0.050*	0.337	0.425
	0.012*	0.026*	0.279	0.024*	0.123	0.421	0.055	0.093	0.003**
	0.124	0.263	0.226	0.086	0.357	0.685	0.177	0.799	1.000
MSCI US Govt Bond Index	0.909	1.000	1.000	0.363	0.721	0.732	0.084	0.323	0.510
	0.602	0.994	1.000	0.384	0.895	0.942	0.258	0.279	0.476
	0.466	0.985	1.000	0.296	0.839	0.845	0.344	0.720	1.000
Lehman US Aggregate Index	0.894	0.998	1.000	0.331	0.506	0.410	0.049*	0.207	0.39
	0.672	0.984	0.997	0.424	0.679	0.742	0.190	0.081	0.987
	0.451	0.956	0.992	0.251	0.573	0.716	0.225	0.711	1.000

Bivariate results

The bivariate results of the Keenan (1985), Tsay (1986) and Teräsvirta *et. al.*, (1993) V23 tests are presented in the following format. In the interest of brevity, the p-values presented in this section are limited to bivariate relationships which are statistically significant as many test results are found to be insignificant. For completeness, the full set of linearity-in-the-mean bivariate test results are presented in the Appendix section of this study. Similar to the univariate results, we report three p-values for each bivariate test, namely, the conventional p-value, the HC p-value and the HAC p-value, respectively.

The key findings from the bivariate tests are consistent with the univariate results which suggest that conventional tests reject the null hypothesis of linearity-in-the-mean due to heteroskedasticity and autocorrelation in the error disturbances in the underlying tests. When the error disturbances of each test are controlled within a HAC framework, the findings reveal that all p-values are insignificant. The results from the bivariate tests provide overwhelming empirical evidence to demonstrate that stock and bond returns are linear-in-the-mean in a bivariate setting.

The bivariate results of the Keenan (1985) test are presented in Appendix A and B. The common feature of the Keenan (1985) test is the pronounced insignificant p-values. These results can be attributed to one of three rationales. First, Granger and Teräsvirta (1993) and Lee *et. al.,* (1993) suggest that the Keenan (1985) test may lack power in detecting unspecified non-linearity in comparison to alternatives such as Tsay (1986) and Teräsvirta *et. al.,* (1993). The second rationale may be attributable to the fact that asset returns may in fact be linear-in-the-mean. The third possible rationale is that the Keenan (1985) test may not be sensitive to the effects of heteroskedasticity and/or autocorrelation. To determine which of these possibilities are valid, we proceed to examine the bivariate linearity-in-the-mean between stock and bond returns with the Tsay (1986) framework.

Table 3: Tsay (1986) Test – Stocks

This table presents the p-values of the Tsay (1986) tests with the stock indices and equity risk factors as the independent variable. This table reports three p-values for each Tsay (1986) test. The first p-value represents the original Tsay (1986) test. The second p-value is the Tsay (1986) test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the Tsay (1986) test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

		Sector	Global	USA	USA	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World Equity Index	S&P500 All Return Index	MSCI USA Equity Index	HML	UMD
Stocks	Global	MSCI World Equity Index	----	0.114	0.061	0.650	0.672
			NA	0.163	0.048*	0.881	0.927
			----	0.143	0.154	0.913	0.958
Stocks	USA	S&P500 All Return Index	0.179	----	0.013*	0.365	0.491
			0.190	NA	0.202	0.566	0.756
			0.429	----	0.437	0.650	0.870
Stocks	USA	MSCI USA Equity Index	0.410	0.020*	----	0.388	0.489
			0.507	0.187	NA	0.598	0.749
			0.641	0.448	----	0.671	0.863
Stocks	USA	UMD	0.674	0.667	0.830	0.015*	----
			0.916	0.889	0.973	0.307	NA
			0.911	0.887	0.966	0.476	----
Bonds	Global	MSCI World plus Em. Sovrgn Bond Index	0.051	0.069	0.076	0.506	0.726
			0.012*	0.029*	0.048*	0.708	0.876
			0.153	0.241	0.236	0.623	0.884
Bonds	Global	J.P.Morgan Global Bond Index	0.225	0.017*	0.018*	0.481	0.328
			0.374	0.010**	0.011*	0.603	0.152
			0.452	0.189	0.179	0.650	0.460
Bonds	Global	Lehman Global Aggregate Index	0.386	0.042*	0.045*	0.471	0.145
			0.565	0.023*	0.027*	0.614	0.023*
			0.596	0.179	0.175	0.632	0.347
Bonds	USA	MSCI USA Govt. Bond Index	0.330	0.039*	0.045*	0.417	0.143
			0.488	0.036*	0.046*	0.579	0.035*
			0.575	0.183	0.190	0.578	0.293

Table 4: Teräsvirta, Lin and Granger (1993) V23 Test – Stocks

This table presents the p-values of the Teräsvirta *et. al.,* (1993) V23 Tests with stock indices and equity risk factors as the independent variable. This table reports three p-values for each V23 test. The first p-value represents the original Teräsvirta *et. al.,* (1993) V23 test. The second p-value is the V23 test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the V23 test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent (HAC) covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable					
		Sector	Global	USA	USA	USA	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World Equity Index	S&P500 All Return Index	MSCI USA Equity Index	HML	SMB	UMD
Stocks	Global	MSCI World Equity Index	----	0.231	0.165	0.814	0.367	0.309
			NA	0.144	0.027*	0.761	0.300	0.030*
			----	0.270	0.292	0.723	0.403	0.592
Stocks	USA	S&P500 All Return Index	0.392	----	0.019*	0.432	0.880	0.493
			0.241	NA	0.385	0.287	0.893	0.011*
			0.462	----	0.510	0.437	0.671	0.628
Stocks	USA	MSCI USA Equity Index	0.657	0.012*	----	0.426	0.945	0.540
			0.561	0.222	NA	0.359	0.952	0.018*
			0.610	0.597	----	0.473	0.839	0.643
Stocks	USA	HML	0.445	0.261	0.244	----	0.045*	0.220
			0.665	0.651	0.701	NA	0.001**	0.000**
			0.709	0.696	0.721	----	0.404	0.618
Stocks	USA	SMB	0.859	0.027	0.020*	0.018*	----	0.018*
			0.899	0.362	0.416	0.650	NA	0.000**
			0.901	0.446	0.493	0.766	----	0.508
Stocks	USA	UMD	0.305	0.682	0.735	0.027*	0.174	----
			0.712	0.851	0.901	0.383	0.655	NA
			0.757	0.850	0.921	0.470	0.670	----
Bonds	Global	MSCI World plus Em. Sovrgn Bond Idx	0.132	0.186	0.190	0.517	0.936	0.927
			0.016*	0.020*	0.046*	0.580	0.911	0.818
			0.419	0.435	0.441	0.646	0.790	0.781
Bonds	Global	J.P.Morgan Global Bond Index	0.034*	0.017*	0.014*	0.514	0.409	0.555
			0.000**	0.000**	0.000**	0.441	0.000**	0.001**
			0.465	0.357	0.347	0.484	0.447	0.496
Bonds	Global	Lehman Global Aggregate Index	0.068	0.042*	0.035*	0.368	0.370	0.285
			0.000**	0.000**	0.000**	0.358	0.000**	0.000**
			0.425	0.331	0.329	0.445	0.458	0.404
Bonds	USA	MSCI USA Govt. Bond Index	0.076	0.060	0.052	0.334	0.407	0.320
			0.008**	0.001**	0.001**	0.415	0.000**	0.001**
			0.454	0.322	0.329	0.477	0.474	0.371
Bonds	USA	Lehman USA Aggregate Index	0.102	0.109	0.096	0.472	0.673	0.587
			0.014*	0.000**	0.000**	0.533	0.006**	0.007**
			0.487	0.401	0.403	0.645	0.485	0.405

Table 3 presents the statistically significant bivariate Tsay (1986) tests whilst the insignificant tests are reported in Appendix C and D of this study. The higher frequency of statistically significant p-values in Table 3 support the view of Granger and Teräsvirta (1993) and Lee *et. al.,* (1993) that the Tsay (1986) test is more powerful than Keenan (1985). Table 3 reveals that the rejection of the null hypothesis of linearity-in-the-mean occurs when stock returns are the independent variable. The Tsay (1986) tests in Appendix D report insignificant p-values when bonds are the independent variable. Despite the rejection of the null hypothesis of conventional Tsay (1986) tests, the HAC p-values in Table 3 reveal that we cannot reject the null hypothesis of linearity-in-the-mean in the bivariate setting. The evidence from the Tsay (1986) tests suggest that stock and bond returns are linear-in-the-mean and that conventional and heteroskedasticity-adjusted tests incorrectly detect non-linearity due to the autocorrelation effects in the error disturbances. To confirm the results of the Tsay (1986) test, we verify these findings with the Teräsvirta *et. al.,* (1993) V23 test.

The Teräsvirta *et. al.,* (1993) V23 tests results in Table 4 (and the full results in Appendix E and F) support the Tsay (1986) hypothesis tests. The key finding from the Teräsvirta *et. al.,* (1993) V23 p-values in Table 4 demonstrates that stock and bond returns are linear-in-the-mean in a HAC bivariate setting. Again, the conventional V23 test reports statistically significant p-values when stock returns are the independent variable. Consistent with the Tsay (1986) results, the V23 tests with bonds as the independent variable report insignificant p-values in Appendix F. The p-values in Table 4 reveal that the Teräsvirta *et. al.,* (1993) V23 test has a tendency to commit Type I errors by over-rejecting the null hypothesis. Table 4 also reports that insignificant p-values in conventional V23 tests, at times, become statistically significant when the HC p-value is estimated. The change in statistical significance can be attributed to the autocorrelation in the test residuals. This can be easily observed in Table 4 where statistically significant p-values are reported with the serially correlated bond returns as the dependent variable. Consistent with the previous findings, the Teräsvirta *et. al.*, (1993) V23 p-values estimated in the HAC framework are all insignificant.

Conclusion

The linearity-in-the-mean condition is an important concept in empirical finance which receives little research attention. In the context of portfolio selection, the linearity-in-the-mean assumption must hold in order for the covariance matrix to be useful in empirical finance applications. If stock and bond returns are not linear-in-the-mean then the decisions made with these empirical models may be subject to model mis-specification. This study contributes to the debate by examining the univariate and bivariate behaviour of linearity-in-the-mean in the two most important asset classes, stocks and bonds.

Of the two forms of linearity (ie. linear conditional mean and constant conditional variance), this study examines the linearity-in-the-mean which is the least researched in the literature. We develop a formal hypothesis testing approach to measure the linearity-in-the-mean of asset returns in both univariate and bivariate settings. Earlier studies and the conventional hypothesis tests reported in this study show that stock and bond returns are non-linear, in both univariate and bivariate settings. However, on closer examination, we discover that many of the stock and bond returns exhibit heteroskedasticity and autocorrelation which contaminate the error disturbances employed in the hypothesis tests. We propose an approach to control these effects by augmenting the linearity-in-the-mean tests from an F-test to a heteroskedasticity and autocorrelation consistent Wald test framework. The findings demonstrate that previous empirical studies have over-rejected the null hypothesis of linearity-in-the-mean due to heteroskedasticity and autocorrelation in the error disturbances in these tests. When these effects are *jointly* controlled in the hypothesis testing regime, this study overwhelmingly demonstrates that stock and bond returns are indeed linear-in-the-mean.

The discovery that stocks and bonds are linear-in-the-mean provides additional insights to the behaviour of these important asset classes. First, the evidence that the conditional mean in stocks and bonds is linear is good news for mean-variance investors making portfolio investment decisions. Second, this study highlights the contamination effects of heteroskedasticity and autocorrelation on the statistical inference of linearity-in-the-mean tests. Researchers who examine linearity-in-the-mean must be able to discriminate and isolate the effects of *both* heteroskedasticity and autocorrelation from the underlying testing regime, corroborating the concerns of Granger and Teräsvirta (1993) and Lee *et. al.,* (1993). This study shows that a failure to isolate these effects will result in the detection of erroneous rejections of linearity-in-the-mean.

The findings from this study provide several directions for future research. An interesting direction would be the estimation of these same tests on finer sampling frequencies such as weekly or daily returns. Second, while stocks and bond returns are linear-in-the-mean, it is worthwhile to consider the linear behaviour of other markets including foreign exchange, commodities and alternative assets. We intend to pursue these interesting research topics in the future.

Appendix A: Keenan (1985) Bivariate Test – Stocks

This table presents the p-values of the Keenan (1985) tests with the stock indices and equity risk factors as the independent variable. This table reports three p-values for each Keenan (1985) test. The first p-value represents the original Keenan (1985) test. The second p-value is the Keenan (1985) test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the Keenan (1985) test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable					
		Sector	Global	USA	USA	USA	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World Equity Index	S&P500 All Return Index	MSCI USA Equity Index	HML	SMB	UMD
Stocks	Global	MSCI World Equity Index	----	0.642	0.470	0.995	0.916	0.996
			NA	0.727	0.417	1.000	0.950	1.000
			----	0.691	0.712	1.000	0.992	1.000
Stocks	USA	S&P500 All Return Index	0.772	----	0.195	0.936	0.999	0.976
			0.768	NA	0.784	0.980	1.000	0.997
			0.946	----	0.949	0.990	1.000	1.000
Stocks	USA	MSCI USA Equity Index	0.955	0.250	----	0.946	1.000	0.976
			0.968	0.763	NA	0.985	1.000	0.997
			0.990	0.952	----	0.992	1.000	1.000
Stocks	USA	HML	0.996	1.000	1.000	----	0.991	0.958
			1.000	1.000	1.000	NA	1.000	1.000
			1.000	1.000	1.000	----	1.000	1.000
Stocks	USA	SMB	0.990	0.460	0.521	0.910	----	0.998
			0.303	0.837	0.896	1.000	NA	1.000
			0.999	0.786	0.839	1.000	----	1.000
Stocks	USA	UMD	0.996	0.996	1.000	0.194	0.480	----
			1.000	1.000	1.000	0.883	0.972	NA
			1.000	1.000	1.000	0.960	0.995	----
Bonds	Global	MSCI World plus Em. Sovrgn Bond Idx	0.423	0.501	0.527	0.979	1.000	0.998
			0.182	0.311	0.415	0.995	1.000	1.000
			0.709	0.828	0.822	0.988	1.000	1.000
Bonds	Global	J.P.Morgan Global Bond Index	0.823	0.215	0.218	0.974	0.991	0.918
			0.923	0.160	0.175	0.985	0.998	0.708
			0.953	0.766	0.751	0.990	0.998	0.956
Bonds	Global	Lehman Global Aggregate Index	0.946	0.381	0.397	0.972	0.991	0.718
			0.980	0.276	0.304	0.987	0.999	0.274
			0.984	0.752	0.746	0.989	0.999	0.909
Bonds	USA	MSCI USA Govt. Bond Index	0.918	0.364	0.393	0.957	1.000	0.710
			0.964	0.355	0.407	0.982	1.000	0.348
			0.981	0.757	0.768	0.982	1.000	0.874
Bonds	USA	Lehman USA Aggregate Index	0.986	0.615	0.647	0.995	1.000	0.908
			0.998	0.691	0.747	0.999	1.000	0.683
			0.999	0.898	0.903	0.999	1.000	0.894

Appendix B: Keenan (1985) Bivariate Tests – Bonds

This table presents the p-values of the Keenan (1985) tests with the bond indices as the independent variable. This table reports three p-values for each Keenan (1985) test. The first p-value represents the original Keenan (1985) test. The second p-value is the Keenan (1985) test adjusted as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the Keenan (1985) test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable				
		Sector	Global	Global	Global	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World plus Em Sovrgn Index	J.P. Morgan Global Bond Index	Lehman Global Aggregate Index	MS USA Govt Bond Index	Lehman USA Aggregate Index
Stocks	Global	MSCI World Equity Index	0.717	0.986	0.987	0.793	0.989
			0.847	0.999	0.995	0.941	0.998
			0.800	1.000	0.998	0.959	0.999
Stocks	USA	S&P500 All Return Index	0.850	1.000	1.000	0.873	0.999
			0.950	1.000	1.000	0.980	1.000
			0.942	1.000	1.000	0.980	1.000
Stocks	USA	MSCI USA Equity Index	0.813	1.000	1.000	0.878	0.999
			0.935	1.000	1.000	0.980	1.000
			0.928	1.000	1.000	0.981	1.000
Stocks	USA	HML	0.897	0.986	0.993	1.000	1.000
			0.975	0.996	0.997	1.000	1.000
			0.976	0.994	0.997	1.000	1.000
Stocks	USA	SMB	1.000	0.999	0.998	0.998	1.000
			1.000	1.000	1.000	1.000	1.000
			1.000	1.000	1.000	1.000	1.000
Stocks	USA	UMD	0.994	0.782	0.880	0.956	0.946
			1.000	0.817	0.781	0.994	0.994
			1.000	0.897	0.914	0.990	0.988
Bonds	Global	MS World plus Em. Sovrgn Bond Idx	----	0.776	0.426	0.722	0.965
			NA	0.948	0.966	0.896	0.992
			----	0.936	0.959	0.924	0.996
Bonds	Global	J.P.Morgan Global Bond Index	1.000	----	0.963	0.799	0.486
			1.000	NA	0.997	0.941	0.607
			1.000	----	0.991	0.984	0.963
Bonds	Global	Lehman Global Aggregate Index	1.000	0.906	----	0.946	0.618
			1.000	0.992	NA	0.998	0.912
			1.000	0.985	----	0.999	0.987
Bonds	USA	MS USA Govt. Bond Index	0.980	1.000	1.000	----	0.902
			1.000	1.000	1.000	NA	0.975
			1.000	1.000	1.000	----	0.994
Bonds	USA	Lehman USA Aggregate Index	0.955	0.972	0.981	0.308	----
			0.998	1.000	1.000	0.720	NA
			0.997	1.000	1.000	0.917	----

Appendix C: Tsay (1986) Test – Stocks

This table presents the p-values of the Tsay (1986) tests with the stock indices and equity risk factors as the independent variable. This table reports three p-values for each Tsay (1986) test. The first p-value represents the original Tsay (1986) test. The second p-value is the Tsay (1986) test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the Tsay (1986) test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable					
		Sector	Global	USA	USA	USA	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World Equity Index	S&P500 All Return Index	MSCI USA Equity Index	HML	SMB	UMD
Stocks	Global	MSCI World Equity Index	----	0.114	0.061	0.650	0.326	0.672
			NA	0.163	0.048*	0.881	0.440	0.927
			----	0.143	0.154	0.913	0.669	0.958
Stocks	USA	S&P500 All Return Index	0.179	----	0.013*	0.365	0.746	0.491
			0.190	NA	0.202	0.566	0.939	0.756
			0.429	----	0.437	0.650	0.909	0.870
Stocks	USA	MSCI USA Equity Index	0.410	0.020*	----	0.388	0.834	0.489
			0.507	0.187	NA	0.598	0.976	0.749
			0.641	0.448	----	0.671	0.959	0.863
Stocks	USA	HML	0.669	0.914	0.881	----	0.591	0.421
			0.921	0.996	0.993	NA	0.864	0.901
			0.938	0.996	0.992	----	0.885	0.841
Stocks	USA	SMB	0.580	0.059	0.074	0.317	----	0.724
			0.860	0.250	0.326	0.897	NA	0.989
			0.812	0.204	0.252	0.909	----	0.986
Stocks	USA	UMD	0.674	0.667	0.830	0.015*	0.064	----
			0.916	0.889	0.973	0.307	0.525	NA
			0.911	0.887	0.966	0.476	0.713	----
Bonds	Global	MSCI World plus Em. Sovrgn Bond Index	0.051	0.069	0.076	0.506	0.871	0.726
			0.012*	0.029*	0.048*	0.708	0.981	0.876
			0.153	0.241	0.236	0.623	0.979	0.884
Bonds	Global	J.P.Morgan Global Bond Index	0.225	0.017*	0.018*	0.481	0.595	0.328
			0.374	0.010**	0.011*	0.603	0.780	0.152
			0.452	0.189	0.179	0.650	0.798	0.460
Bonds	Global	Lehman Global Aggregate Index	0.386	0.042*	0.045*	0.471	0.596	0.145
			0.565	0.023*	0.027*	0.614	0.805	0.023*
			0.596	0.179	0.175	0.632	0.821	0.347
Bonds	USA	MSCI USA Govt. Bond Index	0.330	0.039*	0.045*	0.417	0.837	0.143
			0.488	0.036*	0.046*	0.579	0.973	0.035*
			0.575	0.183	0.190	0.578	0.969	0.293
Bonds	USA	Lehman USA Aggregate Index	0.548	0.104	0.116	0.645	0.839	0.312
			0.783	0.143	0.176	0.832	0.969	0.139
			0.808	0.328	0.338	0.841	0.966	0.322

Appendix D: Tsay (1986) Tests – Bonds

This table presents the p-values of the Tsay (1986) tests with the bond indices as the independent variable. This table reports three p-values for each Tsay (1986) test. The first p-value represents the original Tsay (1986) test. The second p-value is the Tsay (1986) test adjusted as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the Tsay (1986) test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable				
		Sector	Global	Global	Global	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World plus Em Sovrgn Index	J.P. Morgan Global Bond Index	Lehman Global Aggregate Index	MS USA Govt Bond Index	Lehman USA Aggregate Index
Stocks	Global	MSCI World Equity Index	0.148	0.549	0.556	0.194	0.577
			0.261	0.814	0.716	0.417	0.793
			0.215	0.863	0.795	0.470	0.812
Stocks	USA	S&P500 All Return Index	0.242	0.829	0.912	0.267	0.755
			0.441	0.980	0.991	0.565	0.939
			0.418	0.981	0.993	0.569	0.940
Stocks	USA	MSCI USA Equity Index	0.148	0.817	0.896	0.273	0.754
			0.261	0.976	0.987	0.568	0.937
			0.215	0.978	0.989	0.573	0.938
Stocks	USA	HML	0.297	0.548	0.618	0.975	0.981
			0.540	0.735	0.749	0.999	0.999
			0.541	0.693	0.747	0.999	0.999
Stocks	USA	SMB	0.834	0.784	0.717	0.724	0.811
			0.964	0.935	0.861	0.909	0.947
			0.962	0.931	0.868	0.923	0.960
Stocks	USA	UMD	0.625	0.187	0.276	0.415	0.388
			0.862	0.230	0.200	0.696	0.703
			0.885	0.328	0.357	0.644	0.628
Bonds	Global	MS World plus Em. Sovrgn Bond Idx	----	0.182	0.192	0.151	0.442
			NA	0.434	0.497	0.325	0.670
			----	0.403	0.469	0.377	0.730
Bonds	Global	J.P.Morgan Global Bond Index	0.821	----	0.435	0.199	0.065
			0.969	NA	0.750	0.415	0.105
			0.969	----	0.657	0.595	0.485
Bonds	Global	Lehman Global Aggregate Index	0.912	0.310	----	0.387	0.104
			0.993	0.669	NA	0.768	0.352
			0.994	0.603	----	0.817	0.615
Bonds	USA	MS USA Govt. Bond Index	0.508	0.883	0.993	----	0.304
			0.859	0.993	0.999	NA	0.538
			0.858	0.993	0.999	----	0.704
Bonds	USA	Lehman USA Aggregate Index	0.411	0.470	0.514	0.030*	----
			0.770	0.878	0.896	0.159	NA
			0.761	0.860	0.902	0.363	----

Appendix E: Teräsvirta, Lin and Granger (1993) V23 Test – Stocks

This table presents the p-values of the Teräsvirta *et. al.,* (1993) V23 Tests with stock indices and equity risk factors as the independent variable. This table reports three p-values for each V23 test. The first p-value represents the original Teräsvirta *et. al.,* (1993) V23 test. The second p-value is the V23 test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the V23 test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable					
		Sector	Global	USA	USA	USA	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World Equity Index	S&P500 All Return Index	MSCI USA Equity Index	HML	SMB	UMD
Stocks	Global	MSCI World Equity Index	----	0.231	0.165	0.814	0.367	0.309
			NA	0.144	0.027*	0.761	0.300	0.030*
			----	0.270	0.292	0.723	0.403	0.592
Stocks	USA	S&P500 All Return Index	0.392	----	0.019*	0.432	0.880	0.493
			0.241	NA	0.385	0.287	0.893	0.011*
			0.462	----	0.510	0.437	0.671	0.628
Stocks	USA	MSCI USA Equity Index	0.657	0.012*	----	0.426	0.945	0.540
			0.561	0.222	NA	0.359	0.952	0.018*
			0.610	0.597	----	0.473	0.839	0.643
Stocks	USA	HML	0.445	0.261	0.244	----	0.045*	0.220
			0.665	0.651	0.701	NA	0.001**	0.000**
			0.709	0.696	0.721	----	0.404	0.618
Stocks	USA	SMB	0.859	0.027	0.020*	0.018*	----	0.018*
			0.899	0.362	0.416	0.650	NA	0.000**
			0.901	0.446	0.493	0.766	----	0.508
Stocks	USA	UMD	0.305	0.682	0.735	0.027*	0.174	----
			0.712	0.851	0.901	0.383	0.655	NA
			0.757	0.850	0.921	0.470	0.670	----
Bonds	Global	MSCI World plus Em. Sovrgn Bond Idx	0.132	0.186	0.190	0.517	0.936	0.927
			0.016*	0.020*	0.046*	0.580	0.911	0.818
			0.419	0.435	0.441	0.646	0.790	0.781
Bonds	Global	J.P.Morgan Global Bond Index	0.034*	0.017*	0.014*	0.514	0.409	0.555
			0.000**	0.000**	0.000**	0.441	0.000**	0.001**
			0.465	0.357	0.347	0.484	0.447	0.496
Bonds	Global	Lehman Global Aggregate Index	0.068	0.042*	0.035*	0.368	0.370	0.285
			0.000**	0.000**	0.000**	0.358	0.000**	0.000**
			0.425	0.331	0.329	0.445	0.458	0.404
Bonds	USA	MSCI USA Govt. Bond Index	0.076	0.060	0.052	0.334	0.407	0.320
			0.008**	0.001**	0.001**	0.415	0.000**	0.001**
			0.454	0.322	0.329	0.477	0.474	0.371
Bonds	USA	Lehman USA Aggregate Index	0.102	0.109	0.096	0.472	0.673	0.587
			0.014*	0.000**	0.000**	0.533	0.006**	0.007**
			0.487	0.401	0.403	0.645	0.485	0.405

Appendix F: Teräsvirta, Lin and Granger (1993) V23 Tests – Bonds

This table presents the p-values of the Teräsvirta *et. al.,* (1993) V23 Tests with the bond indices as the independent variable. This table reports three p-values for each V23 test. The first p-value represents the original Teräsvirta *et. al.,* (1993) V23 test. The second p-value is the V23 test re-specified as a Wald test employing an adjusted White (1980) heteroskedasticity-consistent covariance matrix. The third p-value is the V23 test re-specified as a Wald test employing an adjusted Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix. * and ** denote statistical significance at the 5% and 1% levels, respectively.

			Independent Variable				
		Sector	Global	Global	Global	USA	USA
Asset Class	Sector	Dependent Variable	MSCI World plus Em Sovrgn Index	J.P. Morgan Global Bond Index	Lehman Global Aggregate Index	MS USA Govt Bond Index	Lehman USA Aggregate Index
Stocks	Global	MSCI World Equity Index	0.148	0.552	0.328	0.430	0.776
			0.070	0.426	0.236	0.387	0.625
			0.206	0.648	0.457	0.360	0.490
Stocks	USA	S&P500 All Return Index	0.247	0.356	0.184	0.540	0.801
			0.190	0.338	0.108	0.480	0.713
			0.325	0.600	0.385	0.448	0.623
Stocks	USA	MSCI USA Equity Index	0.226	0.370	0.199	0.549	0.807
			0.181	0.344	0.118	0.476	0.711
			0.324	0.606	0.399	0.446	0.624
Stocks	USA	HML	0.270	0.681	0.760	0.738	0.770
			0.518	0.746	0.706	0.642	0.520
			0.580	0.670	0.571	0.498	0.443
Stocks	USA	SMB	0.473	0.787	0.846	0.531	0.854
			0.528	0.780	0.733	0.727	0.897
			0.418	0.696	0.603	0.613	0.871
Stocks	USA	UMD	0.757	0.322	0.482	0.264	0.153
			0.796	0.333	0.372	0.554	0.289
			0.821	0.438	0.484	0.641	0.515
Bonds	Global	MS World plus Em. Sov. Bond Index	----	0.361	0.309	0.167	0.667
			NA	0.405	0.401	0.401	0.711
			----	0.578	0.597	0.492	0.787
Bonds	Global	J.P.Morgan Global Bond Index	0.778	----	0.676	0.392	0.155
			0.907	NA	0.817	0.450	0.207
			0.907	----	0.701	0.663	0.530
Bonds	Global	Lehman Global Aggregate Index	0.776	0.582	----	0.300	0.146
			0.884	0.736	NA	0.508	0.328
			0.847	0.601	----	0.790	0.707
Bonds	USA	MS USA Govt. Bond Index	0.715	0.989	0.883	----	0.452
			0.867	0.992	0.965	NA	0.366
			0.838	0.993	0.980	----	0.749
Bonds	USA	Lehman USA Aggregate Index	0.627	0.644	0.739	0.068	----
			0.769	0.891	0.909	0.155	NA
			0.724	0.871	0.917	0.449	----

Footnotes

1. The Bank of International Settlements (ie. BIS (2006)) estimates the value of the world and US debt markets as at 31 December 2005 at US$44,991.7 billion and US$20,554.8 billion, respectively. The World Federation of Exchanges (2006) values the world and US equity market capitalization as at 31 December 2005 at US$40,987.1 billion and US$17,000.8 billion, respectively.

2. Refer to Bollerslev, Chou and Kroner (1992) and Campbell *et. al.,* (1997) for a survey of the literature on nonconstant variance.

3. According to Poshakwale (2002), this type of investigation can be interpreted as a form of random walk hypothesis (RWH) test which examines whether there is a non-linear function which links current and historical asset returns.

4. There is an emerging debate in the literature as to the very existence of non-linearity and the validity of non-linear modelling. For instance, scholars who advocate the use of non-linear time series models include Hamilton (1989), Teräsvirta (1994, 1998), Taylor and Peel (2000) and Taylor, Peel and Sarno (2001). In contrast, Buncic (2006), Chapman and Pearson (2000) and Jones (2003) argue that the presence of non-linearity is due to model mis-specification rather than genuine non-linear behaviour.

5. Desai and Bharati (1998) perform general linearity tests on a variety of US based stock and bond indices only, however, they do not examine world stock and bond returns. A critique of Desai and Bharati (1998) shows that they consider test mis-specification bias caused by heteroskedasticity, however, they do not address test mis-specification caused by autocorrelation.

6. This subtle re-specification of the test for linearity may create a circumstance whereby non-linearity is not detected in the linear functional form proposed in this study, however, non-linearity may exist if lagged variables are employed. This study does not explore the non-linear possibilities with lagged variables as it is outside the typical portfolio decision making process of a mean-variance investor.

7. Granger and Teräsvirta (1993) remind us that the Keenan (1985) test is identical to the Ramsey (1969) RESET test which has been restricted to quadratic terms only and without the multicollinearity problem.

8. Refer to Tsay (2002) for a review of the Keenan (1985) test.

9. Not only are we interested in examining whether stock and bond returns are linear-in-the-mean, this study also considers the Fama and French (1992, 1993) and the Carhart (1997) risk factors also. It is well established in the finance literature that the Fama and French (1992, 1993) and Carhart (1997) risk factors may explain a large proportion of the variation of stock returns. For a comprehensive examination of the linearity-in-the-mean between stock and bond returns, we also include the Fama and French (1992, 1993) and Carhart (1997) risk factors in the analysis. We thank the Kenneth French data library for the data.

10. The SMB and UMD risk factors report the worst minimum monthly returns which reflect the idiosyncratic risk associated with these factors in comparison to the systematic returns of the entire stock market.

11. While previous studies have partially attributed non-linearity to ARCH effects, some neglected non-linearity has remained unexplained in the literature. The results in Table 2 suggest that the unexplained non-linearity captured in previous studies may be the result of autocorrelation in the error disturbances.

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